1 Introduction

The fundamental particles which form the hadronic matter (e.g. protons, neutrons) are the “quarks” (fermions) which interact via “gluons” (bosons) due to the strong interaction. The quarks come in different “flavours”, namely the “up”, “down”, “strange”, “charmed”, “bottom” and “top” quark, but the strong interaction is flavour blind. QCD is a gauge theory based on the unbroken non-abelian SU(3) group, which assigns every quark one of three “colors”. Since there are eight generators of SU(3), there are eight massless gluons carrying a color charge which mediate the strong interactions between the quarks. QCD is an asymptotically free theory, forces between quarks become weak for small quark separations. The asymptotic freedom property of QCD is intimately linked to the non-abelian structure of the gauge group, which on the other hand causes colored gluons to couple to themselves. These self couplings are believed to be responsible for quark confinement, the fact that color charged particles cannot be isolated and hadrons are colorless. Confinement is a consequence of the dynamics at large distances where perturbation theory breaks down due to ultraviolet divergencies and only the lattice formulation of QCD allowed to study the non-perturbative phenomena. The gauge field is put on a discrete space-time lattice, which acts like a cutoff in the perturbative integrals, and the path integral formalism of quantum field theory is applied by using numerical methods. With these tools it was possible to study confinement, and one of the pioneering papers of Mike Creutz [1] was already showing that the potential between static quarks and antiquarks is asymptotically linear rising with the distance. The corresponding constant force, the string tension, is incredible high, around 1 GeV/fm. The origin of this strong force should be found in the properties of the QCD vacuum. This is highly non-trivial, filled with quantum fluctuations and topological excitations which dominate the behaviour of the QCD vacuum at long distance scales. On the other hand the color electric field between quarks and antiquarks has regular flux lines and does not like to enter the stochastically fluctuating QCD vacuum. Therefore, it is energetically favourable to compress the electric flux-lines to a small tunnel between quark and antiquark. Despite intensive efforts over three decades there is no derivation of confinement from first principles nor is there a generally accepted explanation. Candidates for topological excitations responsible for confinement were mainly instantons, abelian monopoles and vortices. Instantons live on a length scale of around 0.2 fm and can therefore contribute only little to the large distance force between heavy quarks [2].

By a transformation to the dual degrees of freedom one can show analytically that confinement in U(1) lattice gauge theory is due to magnetic monopoles. Kronfeld, Schier-
holz, and Wiese [3] devised a method for non-abelian gauge theories to detect monopoles by abelian gauge fixing and abelian projection. The property that an abelian component of the color field can explain the full string tension was shown in [4] and was dubbed Abelian dominance. The monopole confinement mechanism leads to a very nice picture, the dual superconductor model of confinement, where magnetic monopoles and anti-monopoles form a solenoidal current around the electric flux tube between quark and antiquark. But Del Debbio et al. [5] showed that the hypothesis of Abelian dominance in the maximal Abelian gauge, which was known to work for Wilson loops in the fundamental representation, fails for Wilson loops in higher group representations. Such a problem does not appear in the center vortex picture of confinement.

The center vortex model [6] has been proposed as an explanation of confinement in non-abelian gauge theories. Center vortices, quantised magnetic flux lines, compress the gluonic flux into tubes and cause a linearly rising potential at large separations. Numerical evidence has been produced to support this assumption [7], and in addition, simulations have indicated that vortices could also account for phenomena related to chiral symmetry, such as topological charge and spontaneous chiral symmetry breaking (SCSB) [8, 9, 11, 12]. These non-perturbative features of the QCD vacuum are intimately linked to the properties of the low-lying spectrum of the Dirac operator. The Atiyah-Singer index theorem [13] states that the topological charge of a gauge field equals the index of the Dirac operator, while the Banks-Casher relation [14] sets the spectral density of the near-zero modes proportional to the chiral condensate, the order parameter for SCSB. The fundamental problems of investigating chiral symmetry on the lattice have been overcome by the invention of overlap fermions [10]. The overlap operator obeys the Ginsparg-Wilson relation and features an exact chiral symmetry [11]. The asqtad staggered fermions on the other hand just preserve a continuous subgroup of the full chiral symmetry, but they are much less CPU-intensive and one can study the spontaneous breakdown of this remaining lattice symmetry more efficiently.

We study the influence of center vortices on the low-lying eigenmodes of the Dirac operator, in both the overlap and asqtad formulations. In particular we suggest a solution to a puzzle raised some years ago by Gattnar et al. [15], who noted the absence of low-lying Dirac eigenmodes required for chiral symmetry breaking in center-projected configurations. We show that the low-lying modes are present in the staggered (asqtad) formulation, but not for overlap, and we argue that this is due to the absence of a field-independent chiral symmetry on the very rough center-projected configurations for overlap and “chirally improved” fermions.

2 Vortices and chiral symmetry breaking

Concerning chiral symmetry breaking a remarkable result was found by Forcrand and d’Elia [16], removing vortices from lattice configurations leads to restoration of chiral symmetry. Fig. 1 shows the chiral condensate tending to zero for vortex-removed (“Modified”) configurations. Using the chirally improved Dirac operator Gattringer and the Tübingen group [15] have investigated the influence of center vortices on the properties
of the Dirac spectrum. They have shown, see Fig. 2, that the removal of center vortices eliminates the zero-modes and near-zero modes of the Dirac operator implying via the Banks-Casher relation the restoration of chiral symmetry. But it was not understood why the spectra of the center projected configuration has developed a large gap indicating chirally symmetric field configurations. This is a very interesting result. It is up to now the only case where confinement would not lead to chiral symmetry breaking.

Figure 1: Chiral condensate in quenched lattice configurations before (“Original”) and after (“Modified”) vortex removal. From de Forcrand and D’Elia [16].

Figure 2: The 50 smallest Dirac eigenvalues from 10 different configurations are shown in the complex plane. The spectrum for the original ensemble are compared to the spectrum of center-projected and vortex-removed configurations. From Tübingen group [15]
3 Numerical Results

Figure 3: The first twenty overlap Dirac eigenvalue pairs on the Ginsparg-Wilson circle for a $16^4$ lattice at $\beta_{LW} = 3.3$. The center-projected configurations show a four-fold degeneracy. Zero-modes in vortex-removed configurations disappear for antiperiodic boundary conditions.

In Fig. 3 we present the first twenty overlap eigenvalues for a $16^4$ lattice at $\beta_{LW} = 3.3$. There is a big gap around zero for center-projected data, indicating zero chiral condensate. Looking closer at the center-projected eigenvalues one spots only five of the twenty eigenvalues. This indicates a degenerency of four, caused by the real trivial link variables ($\pm I_2$), where the two colors decouple and the eigenvalue equation $D\psi_n = \lambda_n \psi_n$ is invariant under charge conjugation. The vortex-removed data shows four near-zero modes for each chirality, which can be interpreted as real zero modes since they disappear in case of antiperiodic boundary conditions and therefore are irrelevant to $\chi_{SB}$. We speculated that the reason for the large gap in the vortex-only case was connected with the lack of smoothness of center-projected lattices. Of course the overlap operator, in contrast to the chirally-improved operator, does have an exact global symmetry, but the symmetry transformations are gauge-field dependent [18], and only approximate the $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry transformations of the continuum theory for configurations which vary slowly at the scale of the lattice spacing. Center-projected configurations are not even close to smooth, and the Casher argument, relating confinement to $\chi_{SB}$ need not apply. However, the overlap operator should produce a more reasonable answer when applied to a smoother version of the center-projected lattice. Therefore we perform an interpolation between full (gauged) and projected configurations, reducing the angle between the vector representing group element $U_\mu(x)$ in maximal center gauge, and the vector representing the $SU(2)$ center element $Z_\mu(x)I_2$ by some fixed percentage.

\footnote{Given that the Dirac operator has the Wilson or overlap (but not staggered) form. Thus, if $\psi_n$ is an eigenstate with eigenvalue $\lambda_n$, then $C^{-1}\psi_n^*$ is also an eigenstate, with the same eigenvalue [17].}
Figure 4: The first twenty overlap Dirac eigenvalue pairs from a single configuration on a 16^4 lattice, antiperiodic boundary conditions at β_LW = 3.3, for interpolated fields.

unprojected and center-projected lattices. We see that there is no really obvious gap in the partially-projected lattices, even at 85% projection. This agrees with our conjecture that applying the overlap operator to a smoother version of the vortex-only vacuum would give a result consistent with χSB and the Banks-Casher relation. Staggered and asqtad fermions, on the other hand, do not require a smooth configuration to preserve a subgroup of the usual continuum SU(N_f)_L × SU(N_f)_R symmetry, and by the Casher argument [19] one would expect this remaining symmetry to be spontaneously broken by any confining gauge configuration. Indeed, ref. [20] already reported that ⟨ψ̅ψ⟩ > 0 for staggered fermions on a center-projected lattice. Fig. 5 shows the first twenty asqtad eigenvalues, which distribute very differently now. The low eigenmode density (chiral condensate) increases for center-projected compared to full (original) data. Thus, for the asqtad operator, we have found exactly what was expected prior to the results of Gattnar et al.: the vortex excitations of the vortex-only lattice carry not only the information about confinement, but are also responsible for χSB via the Banks-Casher relation. The vortex-removed data develops a central band around Im λ = 0 of eight doubly degenerate eigenmodes per chirality, which are a remanent of the 32 free-field zero modes (four zero modes for each of four “tastes” times two colors), and play no role in χSB. In fact, these modes again disappear using antiperiodic boundary conditions.

4 Conclusions

We find that the thin vortices found in center projection give rise to a low-lying spectrum of Dirac eigenmodes, providing that the chiral symmetry of the Dirac operator does not depend on the smoothness of the lattice configuration. Thus, the vortex excitations of the vortex-only lattice carry not only the information about confinement, but are also responsible for χSB via the Banks-Casher relation. (C.f. [21])
Figure 5: The first twenty asqtad Dirac eigenvalue pairs from a $16^4$ lattice at $\beta_{LW} = 3.3$. The center-projected configurations show no gap around zero. Zero-modes in vortex-removed configurations disappear for antiperiodic boundary conditions.

References


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