Transonic Viscous Inviscid Interactions in Narrow Channels

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Abstract

During this reporting period we used an existing algorithm, [5] to study unsteady as well as steady transonic flows through channels which are so narrow that the classical boundary layer approach fails. As a consequence the properties of the inviscid core and the viscosity dominated boundary layer region can no longer be determined in subsequent steps but have to be calculated simultaneously. The resulting interaction problem for laminar flows is formulated for both perfect and dense gases under the requirement that the channel is sufficiently narrow so that the flow outside the viscous wall layers becomes one-dimensional in the leading order approximation. The latter allows an interpretation of the flow in the core region by means of the theory of one-dimensional transonic inviscid flow through a Laval nozzle while preserving the essential features of the interaction problem associated with the internal structure of pseudoshocks. The sensitivity of a separation bubble caused by a pseudoshock of sufficient strength to perturbations under the condition of choked flow will be demonstrated.

1 Introduction and Problem Formulation

Viscous inviscid interactions of steady or unsteady transonic flows in narrow channels are considered which are triggered, for example, by a shallow deformation of the channel walls, Fig. 1. Using asymptotic analysis for large Reynolds number $Re = \frac{\tilde{u}_\infty L_\infty}{\tilde{c}_\infty} \rightarrow \infty$ Kluwick and Gittler showed for the case of steady flow of a perfect gas that a consistent interaction theory can be formulated in which the flow inside the inviscid core region is almost one-dimensional, [2]. The former theory can be extended to incorporate dense gas effects as well as unsteady effects if the heights $\tilde{H}$ and $\tilde{h}$ of the channel and the surface mounted obstacle are of orders $\epsilon^{(3-2n)/2} L_\infty$ and $\epsilon^{(3+2n)/2} L_\infty$ and if the length $\tilde{\Delta}$ of the obstacle is of order $\epsilon^{3} L_\infty$ with $\epsilon = Re^{-1/(7+2n)}$. Additionally one requires the time to be scaled by $\epsilon^{5-2n} L_\infty/\tilde{c}_\infty$ preserving the slow time scales connected with the longterm behaviour of the system. Here $\tilde{u}_\infty$, $\tilde{c}_\infty$, $\tilde{L}_\infty$, $\tilde{\nu}_\infty$ denote the flow velocity and the speed of sound in the core region just upstream of the local interaction region, a characteristic length associated with the unperturbed boundary layers adjacent to the channel walls and a reference value of the kinematic viscosity. The parameter $n$ refers to thermodynamic properties of the fluid, see discussion of equ. (5).
The interaction region exhibits a triple deck structure. As in classical triple deck theory, e.g. see [3], the role of the main deck is to transfer the displacement effects exerted by the lower deck unchanged to the upper deck and to transfer the resulting pressure disturbances again unchanged to the lower deck. Here, the fluid motion is governed by the (quasi steady) boundary layer equations in incompressible form

\[ \partial_X U(T, X, Y) + \partial_Y V(T, X, Y) = 0, \]
\[ U \partial_X U + V \partial_Y U = -\partial_X P(T, X) + \partial_Y^2 U \]

where \( T, (X,Y), (U,V) \) and \( P \) denote the time, Cartesian coordinates parallel and normal to the freestream direction, the corresponding velocity components and the pressure. All quantities are suitably scaled. The boundary conditions include the no slip condition at the channel walls, the requirement that the undisturbed velocity profile is recovered in the limit \( X \to -\infty \) and a matching condition between lower and main deck for large \( Y \)

\[ Y = S(T, X) : \quad U = V = 0, \quad X \to -\infty : \quad U = Y, \]
\[ Y \to \infty : \quad U = Y + A(T, X), \]

where \(-A(T, X)\) denotes the perturbation of the displacement thickness caused by the interaction process.

The flow in the upper deck is a quasi one-dimensional transonic flow weakly perturbed by the boundary layer displacement. Klüwick, see [1], showed for transonic flows of dense gases in an asymptotically small neighbourhood of the minimum throat area of a nozzle that the perturbation of the mass flux density can be written as a polynomial function of the pressure disturbance, \( G_{(n)}(P; K, \Lambda, N) \), of order \( n \in \{2, 3, 4\} \) where \( K, \Gamma, \Lambda, N \) denote the transonic similarity parameter, the fundamental derivative of thermodynamics, e.g. see [4], and its higher derivatives, see [1]. If \( n = 2 \) and \( \Gamma > 0 \) then the problem formulated in [2] for a perfect gas is recovered; however in the present treatment the case \( \Gamma < 0 \) is admissible too. To describe the thermodynamic properties of a dense gas in the close vicinity of the so-called transition line \( \Gamma = 0 \) confining the \( \Gamma < 0 \) region, see [1], the parameters \( \Lambda \) (case \( n = 3 \)) and \( N \) (case \( n = 4 \)) also have to be taken into account. The relation for the upper deck flow can now be written as

\[ -\partial_T P(T, X) + \partial_X G_{(n)}(P(T, X); K, \Gamma, \Lambda, N) = Q \partial_X A(T, X) \]

where \( Q \) measures the strength of the coupling between lower and upper deck flow.

2 Numerical Method

After introducing the streamfunction \( \Psi(X, Y) \) \( (U = \partial_Y \Psi, V = -\partial_X \Psi) \) and after mapping the halfspace onto a finite region via a suitable coordinate transformation the resulting PDEs are discretized via a finite difference scheme. The lower deck equations are discretized by usage of a Cranck-Nicholson scheme. The strategy has been developed and tested by St. Braun, B. Scheichl and R. Szeywerth from the Institute of Heat Transfer and Fluid Mechanics, TU Vienna and has been described by R. Szeywerth in
his progress report for the ZID ([5]). The existing C code is adapted to incorporate the new interaction law. It makes use of a nonlinear sparse matrix solver PARDISO which is part of the Intel Math Kernel Library IMKL included in the NAG library. The library is linked statically and the executable is sent to the application server *Phoenix Linux Cluster* of the ZID.

The advantages of the described global strategy are:

- additional equations for additional free parameters can be implemented fairly easily which allows parameter studies,
- flow separation can be calculated.

However the resulting system of algebraic equations to solve usually becomes very large as the number of grid points has to be fairly high in contrast to other strategies lacking the above mentioned advantages (e.g. shooting technique).

In the case of steady triple deck solutions the interaction equation governing the upper deck flow reduces to an algebraic relationship which has to be satisfied at some given point thus giving additional restraints for the pressure closing the problem.

In the case of unsteady triple deck solutions the interaction equation is a hyperbolic PDE in one space dimension which is coupled to the solution of the (quasi-stationary) LD fbw problem. The displacement thickness $A(T, X)$ acts as a source term for the upper deck fbw. For the first numerical results a straightforward discretization based on finite difference schemes is used based on the, so far unproven, assumption that the solution to the pressure despite of the hyperbolic nature of the upper deck equation always remains regular due to the regularising effect of the viscous lower deck. The validity of the approach is checked against known time-dependent solutions of the linearized problem. The discretization is based on implicit Euler method for the unsteady term in the interaction law (fully implicit method). At the moment we undertake some efforts to develop a scheme which makes use of the hyperbolic nature of the left-hand side of equation (5).

### 3 Results

In the steady case relation (5) gives the pressure distribution in a Laval nozzle of shape $A(X)$. In contrast to classical Laval nozzle theory the shape of this "viscous" nozzle now is part of the solution of the problem and is caused by the displacement effect of the viscous boundary layers. Fig. 2 shows the pressure and wall shear distribution of such a solution for $G_{(n)} = \text{sign}(K/\Gamma) P + P^2/2$ with $K > 0$ (subsonic upstream fbw conditions) and $\Gamma > 0$. The constriction caused by the nozzle $S(X; \lambda) = S_0(X) = \lambda(1 + \cos(\pi X/L))/2$ with $L = 2$ and $\lambda = 2.01383...$ is large enough to accelerate the fbw from subsonic to supersonic conditions (the point of transition $P = -1$ corresponds to a local extremum of the flux function $G_{(n)}(P)$) and is very close to a critical value above which no steady solutions exist (numerical solutions show that supersonic downstream fbw conditions are reached just for one critical value of $\lambda$ similar to classical Laval nozzle theory). The fbw in Fig. 2 is shocked back to subsonic fbw conditions via a pseudoshock. In the limit $Q \to 0$ this pseudoshock
tends to a normal shock wave, whereas for finite this discontinuity is regularised due to the presence of the viscous lower deck. The strength of the pseudoshock in Fig. 2 is large enough to trigger flow separation. Referring to the terminology of classical Laval nozzle theory the flow of Fig. 2 is choked.

A well known property of choked transonic flows in diffusers is the occurrence of shock wave oscillations where the underlying mechanism is associated with the interaction between the shock wave and an adjacent region of separated flows for which equs. (1) to (5) represent a simplified model based on first principles.

As a first step toward an asymptotic description of this phenomenon we investigate the sensitivity of a separation bubble to perturbations caused by a pseudoshock of sufficient strength under the condition of choked flow. To this end the solution to the steady problem in Fig. 2 is perturbed by a small oscillating hump downstream of the separation bubble. Fig. 3a to Fig. 3c display the numerical results in the region of interest for three different times in case of a small hump located at \( X = 3 \) oscillating with a time period of 0.1. For the numerical solution of the quasi steady boundary layer equations (1) a Cranck-Nicholson discretisation is used. However the unsteady upper deck equation (5) is discretized via a fully implicit Euler scheme with the maximum time step limited to 0.005.

The disturbances caused by the hump starting to oscillate at time \( T = 0 \) and documented here by the disturbances of the wall shear stress are travelling upstream, Fig. 3a. The properties of the separation bubble remain unchanged initially, however, as soon as the disturbances reach the region of flow separation the disturbances are largely amplified within the separation region, Fig. 3b and c. In order to evolve the solution to larger times the time step in the numerical calculation has to be decreased to increasingly smaller and eventually not feasible values resulting in large calculation times. It is expected that a deeper insight into the mechanism of amplification can be gained through the application of bifurcation theory, which will be part of future efforts. Another aspect of future work concerns the construction of steady pseudoshock solutions in case of dense gases which represent regularised versions of admissible normal shocks predicted by inviscid theory, as there are rarefaction shocks, sonic shocks and split shocks, see [1].

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Figure 2:

Figure 3:
References


